Cambridge Assessment International Education

Cambridge O Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

2538130325

ADDITIONAL MATHEMATICS **4037/21**

Paper 2 **May/June 2020**

2 hours

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Blank pages are indicated.

DC (LK) 198171

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series

$$u_n = a + (n-1)d$$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series

$$u_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 Variables x and y are such that, when $\sqrt[4]{y}$ is plotted against $\frac{1}{x}$, a straight line graph passing through the points (0.5, 9) and (3, 34) is obtained. Find y as a function of x. [4]

2 **(a)** Write $9x^2 - 12x + 5$ in the form $p(x-q)^2 + r$, where p, q and r are constants. [3]

(b) Hence write down the coordinates of the minimum point of the curve $y = 9x^2 - 12x + 5$. [1]

3 DO NOT USE A CALCULATOR IN THIS QUESTION.

$$p(x) = 15x^3 + 22x^2 - 15x + 2$$

(a) Find the remainder when $p(x)$ is divided by $x + 1$. [2]

(b) (i) Show that $x + 2$ is a factor of $p(x)$. [1]

(ii) Write $p(x)$ as a product of linear factors. [3]

4 **(a)** In an examination, candidates must select 2 questions from the 5 questions in section A and select 4 questions from the 8 questions in section B. Find the number of ways in which this can be done. [2]

(b) The digits of the number 6 378 129 are to be arranged so that the resulting 7-digit number is even. Find the number of ways in which this can be done. [2]

5 The vectors $\mathbf{a}$ and $\mathbf{b}$ are such that $\mathbf{a} = \alpha\mathbf{i} + \mathbf{j}$ and $\mathbf{b} = 12\mathbf{i} + \beta\mathbf{j}$.

(a) Find the value of each of the constants α and β such that $4\mathbf{a} - \mathbf{b} = (\alpha + 3)\mathbf{i} - 2\mathbf{j}$. [3]

(b) Hence find the unit vector in the direction of $\mathbf{b} - 4\mathbf{a}$. [2]

6 Find the values of k for which the line $y = kx - 7$ and the curve $y = 3x^2 + 8x + 5$ do not intersect. [6]

7 **(a)** Solve the simultaneous equations

$$10^{x+2y} = 5,$$
$$10^{3x+4y} = 50,$$

giving x and y in exact simplified form. [4]

(b) Solve $2x^{\frac{2}{3}} - x^{\frac{1}{3}} - 10 = 0$. [3]

8 **(a)** Expand $(2-x)^5$, simplifying each coefficient. [3]

(b) Hence solve $\dfrac{e^{(2-x)^5} \times e^{80x}}{e^{10x^4+32}} = e^{-x^5}$. [4]

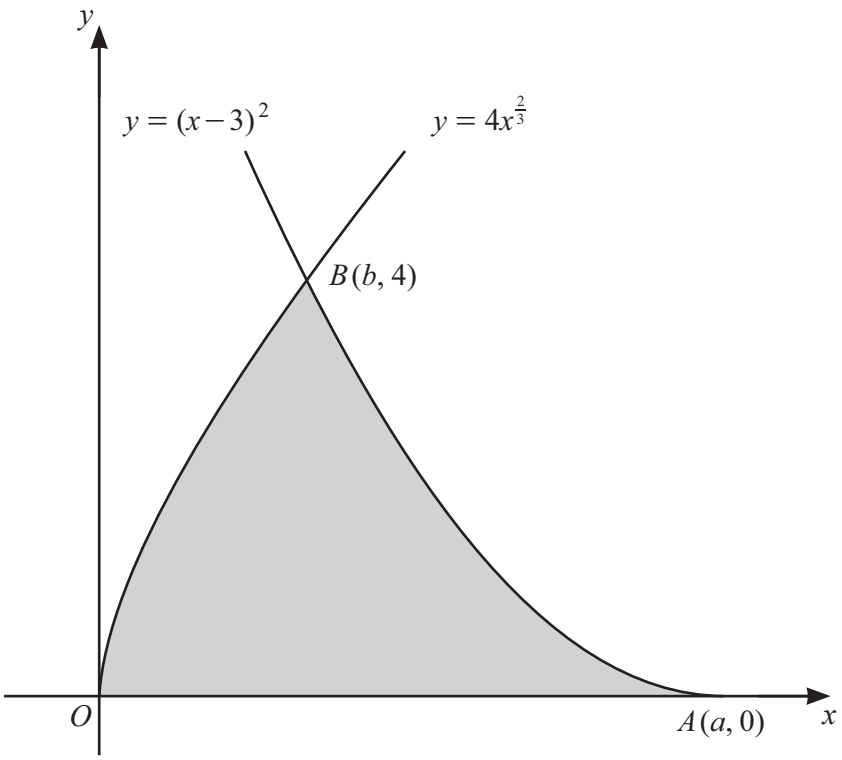
9 A particle travels in a straight line. As it passes through a fixed point O, the particle is travelling at a velocity of $3\,\text{ms}^{-1}$. The particle continues at this velocity for 60 seconds then decelerates at a constant rate for 15 seconds to a velocity of $1.6\,\text{ms}^{-1}$. The particle then decelerates again at a constant rate for 5 seconds to reach point A, where it stops.

(a) Sketch the velocity-time graph for this journey on the axes below. [3]

(b) Find the distance between O and A. [3]

(c) Find the deceleration in the last 5 seconds. [1]

10



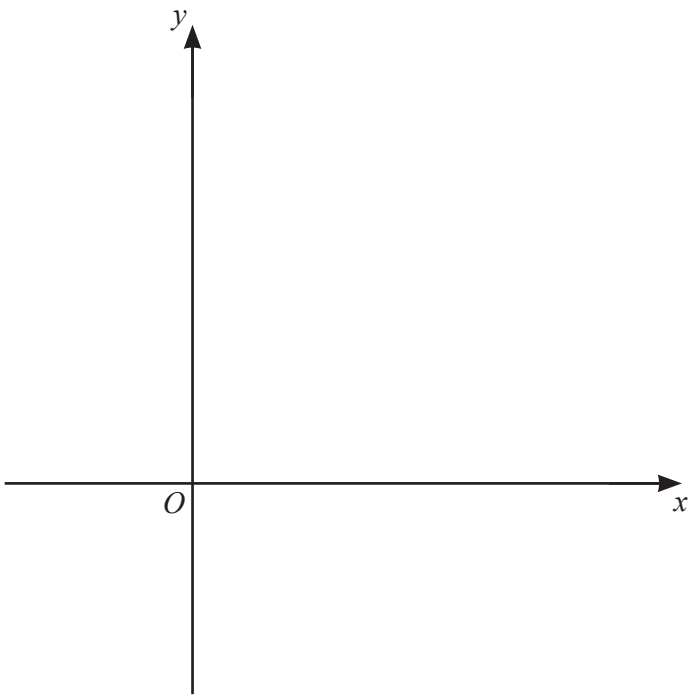
The diagram shows part of the graphs of $y = 4x^{\frac{2}{3}}$ and $y = (x-3)^2$. The graph of $y = (x-3)^2$ meets the x-axis at the point $A(a, 0)$ and the two graphs intersect at the point $B(b, 4)$.

(a) Find the value of a and of b. [2]

(b) Find the area of the shaded region. [5]

11 The function f is defined by $\mathrm{f}(x) = \ln(2x+1)$ for $x \geqslant 0$.

(a) Sketch the graph of $y = \mathrm{f}(x)$ and hence sketch the graph of $y = \mathrm{f}^{-1}(x)$ on the axes below. [3]



The function g is defined by $\mathrm{g}(x) = (x-4)^2 + 1$ for $x \leqslant 4$.

(b) **(i)** Find an expression for $\mathrm{g}^{-1}(x)$ and state its domain and range. [4]

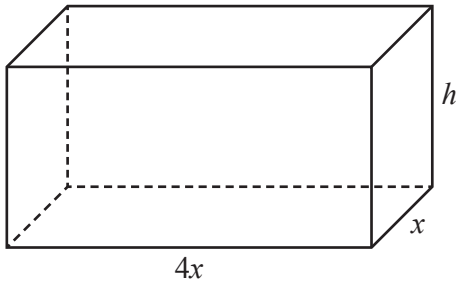
(ii) Find and simplify an expression for $\mathrm{fg}(x)$. [2]

(iii) Explain why the function gf does not exist. [1]

12 **(a)** Find the x-coordinates of the stationary points of the curve $y = e^{3x}(2x+3)^6$. [6]

(b) A curve has equation $y = f(x)$ and has exactly two stationary points. Given that $f''(x) = 4x - 7$, $f'(0.5) = 0$ and $f'(3) = 0$, use the second derivative test to determine the nature of each of the stationary points of this curve. [2]

(c) In this question all lengths are in centimetres.



The diagram shows a solid cuboid with height h and a rectangular base measuring $4x$ by x. The volume of the cuboid is $40\,\text{cm}^3$. Given that x and h can vary and that the surface area of the cuboid has a minimum value, find this value. [5]

BLANK PAGE