Cambridge International AS & A Level

Cambridge Assessment International Education
Cambridge International Advanced Subsidiary and Advanced Level

CANDIDATE NAME

CENTRE NUMBER | | | | | |

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2590531708

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2019**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **20** printed pages.

1 Find the term independent of x in the expansion of $\left(2x + \frac{1}{4x^2}\right)^6$. [3]

2 An increasing function, f, is defined for $x > n$, where n is an integer. It is given that $f'(x) = x^2 - 6x + 8$. Find the least possible value of n. [3]

3 The line $y = ax + b$ is a tangent to the curve $y = 2x^3 - 5x^2 - 3x + c$ at the point $(2, 6)$. Find the values of the constants a, b and c. [5]

4 A runner who is training for a long-distance race plans to run increasing distances each day for 21 days. She will run x km on day 1, and on each subsequent day she will increase the distance by 10% of the previous day's distance. On day 21 she will run 20 km.

(i) Find the distance she must run on day 1 in order to achieve this. Give your answer in km correct to 1 decimal place. [3]

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(ii) Find the total distance she runs over the 21 days. [2]

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5 **(i)** Given that $4\tan x + 3\cos x + \dfrac{1}{\cos x} = 0$, show, without using a calculator, that $\sin x = -\frac{2}{3}$. [3]

(ii) Hence, showing all necessary working, solve the equation

$$4\tan(2x - 20°) + 3\cos(2x - 20°) + \frac{1}{\cos(2x - 20°)} = 0$$

for $0° \leqslant x \leqslant 180°$. [4]

6 A straight line has gradient m and passes through the point $(0, -2)$. Find the two values of m for which the line is a tangent to the curve $y = x^2 - 2x + 7$ and, for each value of m, find the coordinates of the point where the line touches the curve. [7]

7 Functions f and g are defined by

$$\text{f} : x \mapsto \frac{3}{2x+1} \quad \text{for } x > 0,$$

$$\text{g} : x \mapsto \frac{1}{x} + 2 \quad \text{for } x > 0.$$

(i) Find the range of f and the range of g. [3]

(ii) Find an expression for $\mathrm{fg}(x)$, giving your answer in the form $\frac{ax}{bx+c}$, where a, b and c are integers. [2]

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(iii) Find an expression for $(\mathrm{fg})^{-1}(x)$, giving your answer in the same form as for part **(ii)**. [3]

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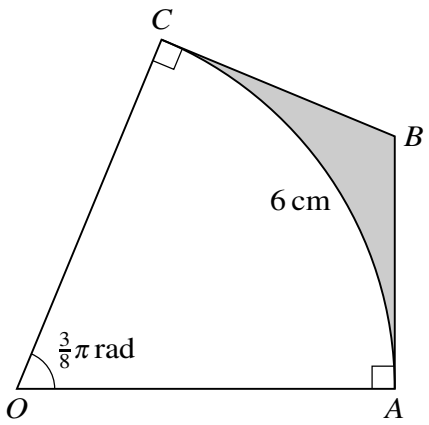

The diagram shows a sector OAC of a circle with centre O. Tangents AB and CB to the circle meet at B. The arc AC is of length 6 cm and angle $AOC = \frac{3}{8}\pi$ radians.

(i) Find the length of OA correct to 4 significant figures. [2]

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(ii) Find the perimeter of the shaded region. [2]

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(iii) Find the area of the shaded region. [4]

9 A curve for which $\frac{dy}{dx} = (5x - 1)^{\frac{1}{2}} - 2$ passes through the point $(2, 3)$.

(i) Find the equation of the curve. [4]

(ii) Find $\frac{d^2y}{dx^2}$. [2]

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(iii) Find the coordinates of the stationary point on the curve and, showing all necessary working, determine the nature of this stationary point. [4]

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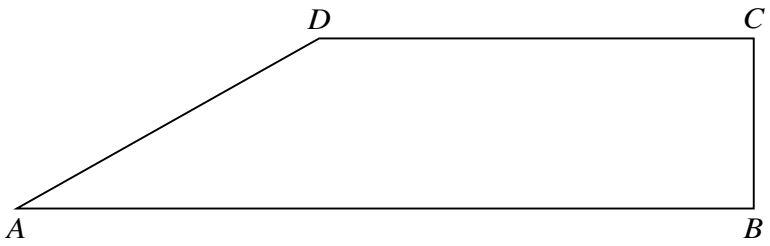

Relative to an origin O, the position vectors of the points A, B, C and D, shown in the diagram, are given by

$$\overrightarrow{OA} = \begin{pmatrix} -1 \\ 3 \\ -4 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix}, \quad \overrightarrow{OC} = \begin{pmatrix} 4 \\ -2 \\ 5 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OD} = \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}.$$

(i) Show that AB is perpendicular to BC. [3]

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(ii) Show that $ABCD$ is a trapezium. [3]

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(iii) Find the area of $ABCD$, giving your answer correct to 2 decimal places. [3]

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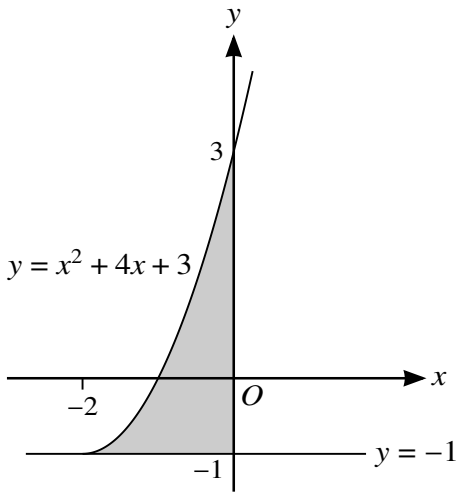

The diagram shows a shaded region bounded by the y-axis, the line $y = -1$ and the part of the curve $y = x^2 + 4x + 3$ for which $x \geqslant -2$.

(i) Express $y = x^2 + 4x + 3$ in the form $y = (x + a)^2 + b$, where a and b are constants. Hence, for $x \geqslant -2$, express x in terms of y. [4]

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(ii) Hence, showing all necessary working, find the volume obtained when the shaded region is rotated through 360° about the ***y*-axis**. [6]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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