MATHEMATICS

Paper 9709/12
Pure Mathematics

Key message

In question **10**, the phrase: 'showing all necessary working, find by calculation' was deliberately inserted so that candidates who simply obtained the area by using the integration function on their calculators with no method shown, received no credit. Many candidates showed their working out which meant that wrong final answers could still receive credit for correct working. All candidates would benefit from following this very important advice as similar questions in the future will very probably have the same pattern of wording.

General comments

The paper proved to be a difficult challenge for many candidates, particularly questions **7** and **9b**. The standard of presentation was generally good with candidates setting their work out in a clear readable fashion with very few candidates dividing the page into two vertically. Most candidates appeared to have sufficient time to complete the paper.

Comments on specific questions

Question 1

This question was generally well answered by most candidates. Some candidates struggled with minus signs and either made both terms positive or both terms negative. Some candidates seemed unfamiliar with the technique required for part **(ii)** and ended up with only one term in x^5 or equated their two terms rather than adding and equating to 0.

Answer: **(i)** 80 and –32 **(ii)** 2/5

Question 2

A large majority of candidates realised the need to integrate but many weaker candidates used the equation of a line formula and received no credit. Some candidates forgot to use + c after integrating and so made no further progress. The number of minus signs involved in substituting –1 also proved challenging for some but many fully correct answers were seen.

Answer: $y = x^3 + x^{-2} + 3$

Question 3

The majority of candidates gained full marks on this question however some weaker candidates struggled to interpret the given information and were unable to form the required equations.

Answer: 93

Question 4

There was a mixed response to part **(a)** of this question. Many candidates knew how to solve the equation but did not give an exact answer as requested in the question. Others were unsure how to start or gave a positive rather than a negative answer. Part **(b)** proved more challenging than expected with many candidates unable to factorise the equation. Attempting to square each term was a common mistake of weaker candidates. Stronger candidates often lost marks due to cancelling and losing one of the solutions or giving answers in degrees rather than radians.

Answer: **(a)** $\dfrac{-\sqrt{3}}{6}$ **(b)** $\dfrac{\pi}{3}, \dfrac{\pi}{2}$

Question 5

Many candidates scored full marks on this question but weaker candidates were sometimes confused by the information presented and seemed unsure of what was expected from them. A sketch may well have helped and candidates should perhaps be encouraged to do one of these for co-ordinate geometry questions. In part **(ii)** some candidates correctly found the point X but did not continue to find the distance BX.

Answer: **(i)** $y - 3 = \dfrac{1}{2}(x - 7)$ **(ii)** 7.33

Question 6

The majority of candidates scored very well on this question although weaker ones struggled in part **(i)** as they were unaware of the formulae for the volume and surface area of a cylinder. In part **(ii)** the vast majority of candidates realised the need to differentiate and set to zero and were able to cope well with the negative power. Some weaker candidates were unsure how to deal with π when differentiating and a greater number were unsure how to confirm that the flask was most efficient at their value of r. Many found the value of A rather than confirming that the area was a minimum.

Answer: **(ii)** 5.4

Question 7

This question proved to be one of the more difficult ones on the paper. In part **(i)** the expected method was to find the length of CA and then deduce that CP was $\dfrac{3}{5}$ of CA. Many candidates struggled with this and some weaker ones used a circular argument with the answers from part **(ii)**. Part **(ii)** was similarly found difficult and although most candidates were very familiar with the method for the scalar product in part **(iii)** quite a few used OP with BP instead of using CP.

Answer: **(ii)** 2.4**i**+1.2**k**, 2.4**i**–2.4**j**+1.2**k** **(iii)** 70.5°

Question 8

Many almost completely correct answers were given to this question but weaker candidates often made no progress in either part. In part **(i)** many of them failed to substitute –2 and –3 into the original expression and so received no credit. The vast majority of candidates however realised the need to do this and then to solve the equations obtained. A number of sign errors occurred and where candidates then solved these incorrect simultaneous equations directly from a calculator then no method marks could be awarded. In part **(ii)** a significant number of candidates did not realise the need to complete the square first in order to find the inverse function and so made no progress. A number of sign errors occurred but those who did complete the square were generally successful, although only the very best candidates realised that due to the given domain the – rather than the + option was required after square rooting.

Answer: **(i)** $a = -2$ or $\dfrac{3}{2}$, $b = 3$ or $\dfrac{5}{4}$ **(ii)** $\dfrac{1}{2} - \sqrt{\left(x + \dfrac{13}{4}\right)}$, $x \geq \dfrac{-13}{4}$

Question 9

Part **(a)** was generally well done but part **(b)** proved to be a significant challenge even for many good candidates. In part **(i)** the vast majority were able to use the angles in a triangle adding up to 180 degrees or 2π radians to prove the given answer. In part **(ii)** a surprisingly large number of candidates failed to use the formula $\frac{1}{2}ab\sin C$ for the area of a triangle. A number of candidates who did find the required area then cancelled the $\frac{1}{2}$ with the 2 in $\sin 2\alpha$. In part **(b)** many candidates realised the link with part **(a)** but were unable to determine the required values of r and α and so were unable to make much progress. Those who found the area of the triangle *ABC* and then took away the 3 equal segments were generally successful.

Answer: **(a)(ii)** $r^2\alpha - \frac{1}{2}r^2\sin 2\alpha$ **(b)** $16\sqrt{3} - 8\pi$

Question 10

This question was generally very well done with many candidates scoring full marks. In part **(i)** a number of candidates failed to realise the significance of the words 'write down' and spent a significant amount of time expanding and attempting to solve the subsequent equation. In part **(ii)** the vast majority of candidates realised the need to differentiate and find the equation of the tangent although some weaker candidates then set the differential equal to 0 rather than substituting in 3.As was indicated in the key message above, the phrase: 'showing all necessary working' was deliberately inserted in part **(iii)** so that candidates who simply obtained the area by using the integration function on their calculators with no method shown, received no credit. Most candidates realised this and attempted to show their working although some simply wrote down the answer from their calculator and showed either no working or completely incorrect working. A number of candidates used the wrong limits after integrating the curve correctly and then took more time than was necessary integrating the equation of the line rather than simply finding the area of the triangle.

Answer: **(i)** $\frac{1}{3}$ **(ii)** $\frac{5}{3}$ **(iii)** $\frac{8}{9}$

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Key messages

Candidates should always ensure that they are working to the correct level of accuracy and giving their answers to the correct level of accuracy as specified on the front of the examination paper, unless told otherwise. They should also ensure that they have answered the questions fully.

General comments

The number of entries for this first time of sitting was relatively small, hence making it difficult to comment fully on the questions and candidate performance. For some questions, an expectation of what was necessary to answer the question fully has been given. There did not appear to be an issue with the length of the paper and the time given to answer the questions.

Comments on specific questions

Question 1

Most candidates made use of algebraic long division as was intended. Problems occurred when attempting to deal with the lack of a term in x, which subsequently lead to errors in the quotient. Correct use of the Remainder Theorem to obtain a remainder was given credit.

Answer: Quotient $2x^2 - x + 2$, Remainder 6

Question 2

Most candidates attempted to solve the problem by squaring both sides of the given inequality, with subsequent work leading to the correct critical values. Obtaining the correct range for x was seldom done correctly with many candidates giving their inequality signs in the 'incorrect' direction. Centres should encourage candidates to check their solutions using a couple of numerical values. A graphical method, inspection or use of 2 equations were also acceptable methods which could have been used.

Answer: $x < -8$, $x > \dfrac{2}{3}$

Question 3

Whilst most candidates were able to deal with the simplification of $2\ln x$ to $\ln x^2$, very few recognised the need to use the addition rule for 2 logarithms and were unable then to obtain the required 3-term quadratic equation. It was expected that candidates would reject the negative solution to this quadratic equation in order to obtain the final accuracy mark.

Answer: $x = \dfrac{3}{2}k$

Question 4

(i) Most candidates do this type of question well, but they must ensure that they are working to the correct number of decimal places as specified in the question and to also give their final answer as a rounded answer to 3 decimal places, not a truncated answer. Unfortunately the final answer of 1.515 was too common.

(ii) Very few correct solutions were seen, with candidates mistakenly using their answer to part **(i)** rather than finding an exact solution to the equation $x = \sqrt{\frac{1}{2}x^2 + 4x^{-3}}$.

Answer: **(i)** 1.516, **(ii)** $\sqrt[5]{8}$ or $8^{0.2}$

Question 5

A correct integration was given by most candidates, together with the correct application of the limits and equating to 65. Few candidates were able to re-arrange the resulting equation correctly, with problems involving logarithms being the cause of most incorrect solutions. Some candidates did not give their final answer to the required level of accuracy as specified in the question.

Answer: 1.097

Question 6

(i) It was important that candidates recognise the need to differentiate the given equation as a product in order to proceed. Quite a few correct solutions were seen with candidates correctly substituting $x = 0$ into their gradient function to subsequently obtain the correct equation for the tangent.

(ii) Usually done correctly by those candidates who attempted this part, many candidates found the x coordinate as required but omitted to find the corresponding y coordinate; thus the prompt for candidates to ensure that they have completed a question completely before moving on.

Answer: **(i)** $y = 6x$, **(ii)** $(0.554, 1.543)$

Question 7

(i) Most candidates recognised the need to differentiate the given equation implicitly, however some failed to deal with 24 correctly whilst other introduced a spurious $\frac{dy}{dx}$ into their equation. Many correct derivatives were seen, but candidates were unable to justify why the gradient of the curve was never positive.

(ii) Many candidates were able to obtain the correct relationship between x and y , with many obtaining the solution $(2,2)$. No attempts to gain the second solution were seen. Candidates should be alert to the fact that equations of this type often have more than one solution.

Answer: **(i)** $\frac{dy}{dx} = -\frac{6x^2}{3y^2}$, **(ii)** $(2,2)$ and $(2.88, -2.88)$

Question 8

(i)   Very well done by the great majority of candidates, recognising the need to obtain both of the terms on the left hand side of the identity in terms of sine and cosine first.

(ii) (a)   Provided candidates realised the need to use the double angle formula most were able to obtain the given expression in a form that would enable them to find the least possible value. However, once in this form, candidates did not recognise that the least value of the trigonometric term was zero and thus the least value of the expression was the value of the constant term obtained.

(b)   Candidates were expected to obtain the integrand $\frac{1}{2}\sec^2 2x$, having made use of the identity in part **(i)**.

Answer: **(ii)(a)** 3, **(b)** $\frac{1}{4}\sqrt{3} - \frac{1}{4}$

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General comments

In general the presentation of the work was acceptable and most candidates attempted all the questions. It was a concern to see that many candidates had not taken on board the comments in the 2014 and 2015 reports. Namely that when attempting a question it is essential that sufficient working is shown to indicate how they arrive at their answer, whether they are working towards a given answer or an answer that is not given. For example question **9(ii)**, clearly showing the substitution of their limits, or question **2**, clearly displaying the substitution of their values in the quadratic equation solution formula.

Candidates should realise that where they have made an error, as often happened in question **2**, then it is even more essential to show the details of the solution of their quadratic equation and not just two incorrect answers from their calculator. The latter approach will result in the method mark for the solution to this equation being withheld. Once again the reason for this decision is that the examination is a mathematics examination, not a calculator examination.

Where numerical and other answers are given after the comments on individual questions it should be understood that alternative forms are often acceptable and that the form given is not necessarily the only 'correct answer'.

Comments on specific questions

Question 1

The solution to this question required, (i) application of the laws of logarithms for product/quotient and power, (ii) reduction of a logarithmic equation to an equation free of logarithms by taking the inverse, (iii) solution of a two term quadratic to obtain two roots then (iv) the selection of only the exact positive root, based on knowing that the logarithmic function is undefined for negative values. Candidates who only produced the positive solution were awarded full marks, however it would have been better to have seen the negative root and for this to have been rejected, rather than wonder whether the candidate had failed to realise that there was another solution to the quadratic equation.

Answers: $x = \dfrac{2}{\sqrt{3}}$

Question 2

This question required the use of the correct $\tan(A \pm B)$ formulae. These were usually correctly applied, but occasionally the signs were incorrect, for example both positive or both negative. Some of the candidates then made errors in removing the denominators to produce a correct un-simplified horizontal equation. Such an error meant that only the method mark was still available to score. Unfortunately, as mentioned in the general comments, some candidates even failed to gain this mark by solving the incorrect quadratic equation on their calculator.

Answers: θ = 28.7° and 165.4°

Question 3

This proved to be a question where candidates could score most of the marks.

(i)   It was not sufficient to state f(1) < 0 and f(2) > 0, the actual values were required. However, even f(1) = − 5 and f(2) = 8, did not score both marks since it was necessary to complete the argument with a statement that there was a change of sign, and hence a root of the equation between 1 and 2.

(ii)   Some candidates opted to work from the rearranged equation to reach the given equation and this was perfectly acceptable.

(iii)   Far too many candidates rounded incorrectly at the end and gave the final answer as x = 1.77.

Answers: **(ii)** 1.78

Question 4

(i)   Various successful approaches were seen, for example substituting $x = -\dfrac{1}{2}$ and equating to zero; dividing by (2x +1) and equating remainder to zero; or showing that factorisation produced the specific value of a, in fact the latter approach also produced the solution to **(ii)(a)**.

(ii) (a) Since the value of a was virtually always correct candidates had little difficulty in factorising p(x).

(b)   Most candidates obtained the correct answer, but often failed to justify this and so lost marks. The solution required much more than just stating that the quadratic factor was always positive. Some candidates correctly showed that the quadratic factor had a constant sign, either by using the discriminant or by actually finding the complex roots of the quadratic equation. However, this was still insufficient since it is essential to show that the quadratic term is actually positive. This could have been achieved by evaluating the quadratic factor at any value of x or by considering the coefficient of the x^2 term.

Answers: **(i)** a = 3 **(ii)(a)** $2x^2 - x + 2$ **(ii)(b)** $x > -\dfrac{1}{2}$

Question 5

(i)   dx/dθ was usually correct, but sometimes √3 was omitted. Again with a given answer all the detailed working was required, for example substitution to obtain 1 + tan²θ, followed by its replacement by sec²θ and the appropriate cancellation of terms, in order to gain full marks.

(ii)   Most candidates used the double angle formula, but sign errors were common, as they were in the integration process. Too often integration had either sinθ instead of sin2θ or the omission of 2 from the denominator.

Answers: $\dfrac{1}{12}\sqrt{3}\,\pi + \dfrac{3}{8}$

Question 6

(i)   Some candidates ignored the differentiation of any term involving y. However those candidates that realised that implicit differentiation, together with the product rule, was required were usually successful apart from the occasional sign error. Other good solutions were by candidates obtaining an expression for sin$y$ before commencing their differentiation.

(ii)   Provided candidates had only made a small sign error in **(i)** they were able to solve for ln$x$ and gain the first two method marks. However some candidates believed that setting $\dfrac{dy}{dx}$ to zero required both the numerator AND the denominator to be set to zero.

Answers: **(i)** $\dfrac{dy}{dx} = \dfrac{1 + \ln x}{(2 + \ln x)^2}$ **(ii)** $x = \dfrac{1}{e}$

Question 7

(i) Separation of variables was required in order to make any progress with this question. However, the candidates who managed to undertake this often experienced difficulty with the integration of $\dfrac{1}{e^x}$. Common to see either this converted to e^x or e^{-x} incorrectly integrated as e^{-x}. Although the integration of xe^x was usually correct.

(ii) As in the final part of Q6, unless the result of the previous section was virtually completely correct then it was difficult to score this mark.

Answers: **(ii)** $y = - \ln(e^x(1 - x))$

Question 8

(i) Most candidates successfully showed that the vectors $2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ and $2\mathbf{i} - \mathbf{j} - \mathbf{k}$ were perpendicular, but omitted to test that no point on the line lies in the plane.

(ii) Few candidates made much progress with this part, incorrectly using either $\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ or $5\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ in trying to establish the direction of the line *m*. Only candidates who opted to determine the vector product of $2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ and $2\mathbf{i} - \mathbf{j} - \mathbf{k}$ were really successful.

Answers: **(ii)** $\mathbf{r} = 5\mathbf{i} + 3\mathbf{j} + \mathbf{k} + \mu(\mathbf{i} + 4\mathbf{j} + 2\mathbf{k})$

Question 9

(i) Many fully correct answers, but also lots of arithmetical errors. Too many candidates wasted time in first opting to find the rational fraction of f(*x*) – *A*. This un-necessary working usually lead to more arithmetical errors.

(ii) Unless candidates had found the constant *D* in **(i)** to be zero their integration of the term $\dfrac{C + Dx}{x^2 + 2}$ proved difficult. However, this was only penalised in the final accuracy mark. The expression $A + \dfrac{B}{x}$ was nearly always integrated correctly however the coefficient of the $\ln(x^2 + 2)$ term was often incorrect. To obtain the final mark with the answer given it was necessary to see the clear substitution of the limits and ALL the SUBSEQUENT logarithm work. Omission of steps in this logarithm work, for example $-4\ln 2 + 2\ln 6 - 2\ln 3$ to answer given is NOT acceptable.

Answers: **(i)** $A = 3$, $B = - 4$, $C = 4$, $D = 0$

Question 10

(a) Few candidates made any progress with this question after they had substituted for z and z*, since they did not realise that they had to take real and imaginary parts and solve the resulting equations. Of the few candidates who did know what to do, often they then had arithmetical errors in their solution.

(b) (i) The circle inequality was often correct, but again far too many candidates had either the incorrect centre or a radius which was not equal to 1. The Im z $\geqslant$ 3 statement was usually incorrectly represented by a circle of radius 3 centred at the origin.

   (ii) Only those with the correct diagram in **(b)(i)** could really attempt this section.

Answers: **(a)** $z = \dfrac{1}{3} - \dfrac{2}{3}i$ **(b)(ii)** 0.588 radians or 33.7°

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General comments

The paper was generally well done by many candidates although as usual a wide range of marks was seen. The presentation of the work was good in most cases.

Some candidates lost marks due to not giving answers to 3sf as requested. Marks were also lost due to prematurely approximating within their calculations leading to the final answer. This was particularly noticeable in questions 2, 3, 4 and 5 where exact trigonometric values were given and hence there was no need to evaluate the angles. If angles were evaluated in these cases this lead immediately to a loss of accuracy.

Students should also be reminded that if an answer is required to 3sf, as is the rubric on this paper for non-exact answers, then their working should be performed to at least 4sf.

One of the rubrics on this paper is to take $g = 10$ and it has been noted that virtually all candidates are now following this instruction. In fact in some cases it is impossible to achieve the correct given answer unless this value is used.

Comments on specific questions

Question 1

Since it is not given in the question that acceleration is constant, the best approach here was to use the Work/Energy principle. Most candidates correctly evaluated the gain in kinetic energy and the work done against the resistive force. The total work done by the cyclist is the sum of these two values. However, some candidates found the difference between the values and hence lost the final mark.

For those candidates who assumed constant acceleration, it was necessary to find this acceleration which most performed successfully and then apply Newton's second law to find the driving force. Again several candidates made sign errors when applying this method. In order to find the work done this force is then multiplied by the distance travelled to give the result. Although it was not given that the acceleration was constant, candidates applying this method were given the benefit and allowed to score full marks in this case. However, it is important to read the questions carefully as it may be that only an energy method could be used in some similar cases.

Answer: Total work done by the cyclist = 5940 J (to 3sf)

Question 2

(i)   As the speed of the car is given in the question as constant, then the total force acting on the car is zero and so the driving force is exactly balanced by the resistive force and hence driving force = *F* = 1350 N. Candidates then had to apply the equation *P* = *Fv* in order to find the rate at which the engine was working. Most candidates performed well on this part, although some lost a mark by not giving the answer in the required form. Again it is well worth reading the question carefully.

Answer: The rate at which the engine of the car is working = 43.2 kW

(ii) In this part the car travels up a hill. The sine of the angle is given and this can be used without finding the angle itself. The car is still travelling at constant speed but now the driving force, F, is balanced by the resistive force plus the component of the weight of the car. Some candidates only included one of these forces here. Some also lost marks by forgetting to write 76.5 kW as 76500 W when using $v = P/F$ to find the speed in ms^{-1}

Answer: The constant speed up the hill = 30 ms^{-1}

Question 3

(i) The majority of candidates scored well on this question. Most resolved the forces horizontally (two of the forces having horizontal components) and vertically (all three forces having a vertical component). A few candidates resolved along the directions of the 30 N and 40 N force which was a perfectly correct method. In order to find the magnitude of the resultant in either case the square root of the sum of the squares of the two components was needed. An inverse trigonometric calculation was needed to find the required direction. Care had to be taken if the directions of the 30 N and 40 N forces were used rather than the horizontal and vertical. Many candidates correctly found the answer to both parts.

Answers: Magnitude of the resultant = 31.6 N Direction of the resultant = 18.4° with the positive *x*-axis

(ii) In this part all that was needed was to note that the net force in the vertical has to be zero and since the vertical component had been found by most candidates as 10 N in part **(i)** then by replacing the 50 N force by one of 40 N would produce the required result.

Answer: P = 40

Question 4

(i) Most candidates resolved horizontally to find that $F = 5\cos\alpha$. It was then necessary to resolve vertically to find R from the equation $R + 5\sin\alpha = 8$. Although most candidates correctly found F many assumed that R was merely mg and so wrongly used $R = 8$. The required coefficient of friction can be then found using $F = \mu R$.

Answer: μ = 0.8

(ii) In this part many candidates did not realise that it was necessary to recalculate the normal reaction due to the new force of 10 N that was now acting. Some also misread the question and added an extra 10 N to the force and continued with the question using a force of 15 N. In order to find the acceleration it was necessary to use Newton's second law applied to the particle using the value of μ found in part (i) with the new reaction found in part (ii) to determine the friction term.

Answer: The acceleration of *P* is 8 ms^{-2}

Question 5

(i) The most straightforward approach to finding the acceleration in this part of the question is to apply Newton's second law to the system of car and trailer. This does not involve the tension in the cable which joins them. The alternative approach is to write down the equation of motion for the car and for the trailer separately and solve the resulting simultaneous equations in *T*, the tension in the cable, and *a*, the acceleration of the system. In order to find *T*, one of these equations has to be used. Many candidates were confused as to which forces were acting on each part of the system. A clear force diagram showing the forces acting on the car and another for the trailer would help greatly in problems such as these.

Answers: The acceleration of the system $a = \dfrac{1}{8} = 0.125$ ms^{-2}. The tension in the cable *T* = 1050 N

(ii)   In this part again the best method was to apply Newton's second law to the system as only the new acceleration of the system was needed. Many candidates assumed that the acceleration was unchanged when the driving force was removed whereas it takes a constant negative value until the system comes to rest. Once the acceleration was found one of the constant acceleration equations could be used to find the required time. Most candidates used the correct method to find the time taken but many used an incorrect value for acceleration.

Answer: The time before the system comes to rest is $\dfrac{80}{3} = 26.7$ seconds

Question 6

(i)   This question was well done by many candidates. Horizontal resolution of forces acting on *A* is needed and vertical resolution of forces acting on *B*. Most candidates correctly resolved at *B* but a few wrongly included the weight component when resolving horizontally at *A*. Most correctly found the acceleration of the system and then used $s = ut + \frac{1}{2}at^2$ with $u = 0$ and $s = 2.5$ to find the required time.

Answer: The time taken for *A* to reach the pulley is $\dfrac{1}{2}\sqrt{10} = 1.58$ seconds

(ii)   In this part the frictional force acting on *A* had to be found from the given value of the coefficient of friction. The equation of motion for *B* was unchanged. Most candidates found the friction force correctly and applied it to the equation at *A*. An error that was often seen was to wrongly use the same tension as was found in part (i) rather than eliminate the new tension and find the acceleration *a*. Once *a* was found most candidates correctly chose to use the constant acceleration formula $v^2 = u^2 + 2as$ with $u = 0$ and $s = 2.5$ to find the required speed.

Answer: The speed of *A* immediately before it reaches the pulley is $\sqrt{6} = 2.45\,\text{ms}^{-1}$

Question 7

(i)   This part was correctly found by many who realised that they had to use the fact that the velocity is continuous either at $t = 4$ or at $t = 14$ to find the value of *k*.

Answer: $k = 40$

(ii)   Candidates needed to produce a *v–t* graph showing the three parts of the motion correctly and many made a good attempt at this. The first part of the motion from $t = 0$ to $t = 4$ caused most problems. It should be shown as a quadratic curve starting at the origin, passing through the value of $v = -5$ at $t = 1$, crossing the *t*-axis at $t = 2$ and taking a value of $v = 40$ at $t = 4$. Many wrongly showed this as a straight line from the origin to (4,40) or showed it as a series of straight lines rather than a curve and hence lost the mark. The second part of the motion was represented by a horizontal line from (4,40) to (14,40) and was correctly shown by most candidates. The final part was a straight line with negative slope from (14,40) to (20,28). Some wrongly showed this part as a line which returned to the *t*-axis at $t = 20$. Most candidates scored 2 out of 3 on this part. When drawing a sketch of a *v-t* graph or similar, the axes should be well annotated at all of the key points in order to score full marks.

Answers: A quadratic curve from (0,0) passing through (1,–5), (2,0) and (4,40)
A straight line from (4,40) to (14,40)
A straight line from (14,40) to (20,28)

(iii)   This part could be answered directly from the *v-t* graph by spotting that the acceleration is positive only from $t = 1$ up to $t = 4$. Alternatively the expression for *v* could be differentiated to find the acceleration and in the first region this gives $a = 10t - 10$ which shows that *a* is zero at $t = 1$.

Answer: $1 < t \leqslant 4$

(iv) Almost all candidates found difficulty with this question. Since the question asked for the total distance travelled, great care had to be taken over the treatment of the first 4 seconds of motion. Most candidates integrated the quadratic expression from $t = 0$ to $t = 4$ which gives the displacement at $t = 4$ **not** the distance travelled due to the fact that between $t = 0$ and $t = 2$ the integral is negative. Between $t = 4$ and $t = 20$ the distance travelled can be evaluated directly from the area under the graph and most candidates found this part correctly. If $A = \int_0^2 v \, dt$, $B = \int_2^4 v \, dt$ and C is the area under the v–t graph from $t = 4$ to $t = 20$ then the total distance travelled is given by $- A + B + C$

Answer: The total distance travelled by P in the interval $0 \leqslant t \leqslant 20$ is 644 m

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General comments

This paper cannot be compared with a previous one as this is the first time a March paper has been set. If compared to last November's paper or last June's paper then the standard was very similar.

Most candidates are now using g = 10 as instructed on the front page of the question paper.

The easier questions were **1**, **2(ii)**, **4**, **5(i)** and **6(i)**.

The harder questions were **2(i)**, **3**, **5(ii)**, **5(iii)** and **7**.

The hardest question proved to be number **7**.

Comments on specific questions

Question 1

Many candidates used $\tan\theta = y/x$ instead of $\tan\theta = v_y/v_x$, where θ is the angle of projection.

If V is the speed of projection and θ the angle of projection, then 2 equations can be found by resolving horizontally and vertically. These equations are $V\cos\theta = 15$ and $V\sin\theta = 20$.
By squaring and adding $V = 25$ and by dividing $\tan\theta = 20/15$, $\theta = 53.1$.

Answer: Initial speed = 25 m s^{-1}, Angle of projection = 53.1°

Question 2

(i)   Few candidates were able to do this part of the question. It was necessary to take moments about the point of contact of the hemi-sphere with the plane. The problem was finding the perpendicular distance of *P* from this point. This distance was $0.8 - 0.8\cos\theta$.

(ii)   This part was quite well done. It was done by applying $F = \mu R$, where μ = coefficient of friction.

Answers: **(i)** *P* = 8.75 **(ii)** Coefficient of Friction = 0.146

Question 3

This question proved to be quite difficult.

Let V be the speed of the particle at the 2 points when the particle makes an angle of 45° with the horizontally. The horizontal speed is unchanged and so $V\cos45 = 9\cos60$. This leads to $V = 9\sqrt{2}/2$. Using the vertical motion between the 2 points leads to $-V\sin45 = V\sin45 - gt$.
This results in $t = 0.9$. Finally by using horizontal motion s = $9\cos60 \times 0.9 = 4.05$ m.

Answer: Distance = 4.05 m

Question 4

This question was generally well done.

(i)　　Candidates were required to take moments about both *BC* and *BAG*.

(ii)　　Most candidates gave the correct answer of 45°.

(iii)　　A few candidates found the angle with the horizontal instead of the vertical.

Answers: **(i)** *h* = 0.775, *v* = 0.775 **(ii)** 45° **(iii)** 66.7°

Question 5

(i)　　Most candidates scored both available marks.

(ii)　　Only a few candidates were able to solve this part of the question. A 5 term energy equation was needed in order solve this part. Too often either 1 or 2 terms were omitted.
If *d* is the distance below the equilibrium position, when the speed is 3.5 m s^{-1}, then the equation is $0.6 \times 4.5^2 / 2 + 24 \times 0.2^2 / (2 \times 0.8) + 0.6gd = 0.6 \times 3.5^2 / 2 + 24(d + 0.2)^2 / (2 \times 0.8)$. This leads to *d* = 0.4 and so *AP* = 0.8 + 0.2 + 0.4 = 1.4 m.

(iii)　　Only a few candidates scored any marks on this part of the question. This time a 4 term energy equation was required, as follows:
$24 \times 0.2^2 / (2 \times 0.8) + 0.6 \times 4.5^2 / 2 = 0.6 v^2 / 2 + 0.6g \times 0.5$, where v is the required speed.

Answers: **(i)** Extension = 0.2 m **(ii)** *AP* = 1.4 m **(iii)** Speed = 3.5 m s^{-1}

Question 6

(i)　　This part of the question was usually well done.

(ii)　　Most candidates attempted to integrate to find v. Some candidates arrived at *v* = f(*x*) and not v^2 = f(*x*).

(iii)　　The value of *x*, where *P* comes to rest, was attempted by equating v to zero.

Answers: **(i)** $2v dv/dx = 10 - (\sqrt{3}) x^2$ **(ii)** Maximum speed of *P* = 4(.00) m s^{-1} **(iii)** *x* = 4.16

Question 7

This question proved to be the most difficult question on the paper.

(i)　　Too many candidates mixed up the trigonometric ratios when attempting to resolve vertically. The result should be *R*cos60 + *T*cos30 = 0.2g.

(ii)　　An attempt to use Newton's Second Law horizontally was often used. Unfortunately the radius used was 0.6 instead of 0.6sin60.

(iii)a　　Candidates realised that they needed to solve the 2 simultaneous equations already found. Unfortunately the 2 equations were often incorrect.

(iii)b　　Only a few candidates realised that the greatest value of *T* was when *R* was zero. An attempt at the speed was seen but the wrong radius was again used.

Answers:　**(i)** $R + T\sqrt{3} = 4$ **(ii)** $T - R\sqrt{3} = 0.12 \omega^2 \sqrt{3}$ **(iii)a** *R* = 0.6
　　　　(iii)b Maximum *T* =2.31 and speed = 1.73 m s^{-1}

MATHEMATICS

Key messages

To do well in this paper, candidates must work with 4 significant figures or more in order to achieve the accuracy required.

Candidates should be encouraged to show all workings. In the event of a mistake being made, credit can be given for the method. A wrong answer with no workings scores no marks. A number of candidates did not show sufficient working to make their approach clear and were unable to gain full credit.

When drawing graphs, candidates should use sensible scales that enable accurate readings to be achieved and label axes including units.

General comments

It was pleasing that many of the candidates who took this paper had a good knowledge of the syllabus. The paper allowed candidates to demonstrate their knowledge of basic skills.

Comments on specific questions

Question 1

The use of the coded mean appeared to cause some difficulty for candidates.

(i)   The majority of candidates who attempted the question recognised that all that was required was to multiply the mean by the number of values. A number chose to calculate the value of the constant a initially and then solved $\Sigma(x - a) = 362$

(ii)   Good solutions expanded $\Sigma(x - a)$ as $\Sigma x - \Sigma a$, substituting their answer from **(i)** into the equation before solving. Weaker solutions failed to replace Σa with $10a$, and thus solved an incorrect equation. An alternative approach used was to consider a single term $x - a$, which was then equated with 36.2.

Answer: **(i)** 862 **(ii)** 50

Question 2

This was a standard 'probability without replacement' question. Good solutions often included a tree diagram to help identify required outcomes, with the best recognising that there were only 2 options necessary (white or non-white roses). The alternative approach of considering combinations of roses was successfully used by many. A common error was to ignore choosing the third rose as there were only 2 white roses available. There requirements of the probability distribution table were understood, but an attempt at the probabilities was necessary before credit was available. Solutions were seen where candidates used the fact that the total probability was 1 to calculate the third probability.

Answer: P(0) = 7/15, P(1) = 7/15, P(2) = 1/15

Question 3

Candidates should be aware that not all dice have 6 sides, as a number of solutions were presented which ignored the 7 and 8. It was unfortunate that a few candidates did not show any workings in this question as they stated incorrect probabilities which may have come from undertaking processes which would have gained credit.

(i)   The best solutions drew an outcome space and identified the outcomes that fulfilled the requirement of *R*. Many solutions listed just the 5 different pairs of values that were possible. Weaker solutions did not recognise that the values could be reversed, and so did not double the probability.

(ii)   The best solutions used the outcome space constructed for **(i)** to identify the necessary terms. There was little confusion in understanding the meaning of 'product' although the value was not always calculated accurately. A number of solutions were seen where inaccurate counting of correctly marked terms occurred.

(iii)   Most candidates recognised that they needed to consider P($R \cap S$) to determine independence. Good solutions stated the requirements for independence, used the previous parts within their arguments, stating clearly each calculation and reaching a final conclusion. A number of circular arguments were presented, where P(*R*) x P(*S*) was calculated and then stated as equal to P($R \cap S$). Errors made in **(i)** were often repeated in this part, but appropriate comparisons were valid. Candidates should be aware that they are expected to state a conclusion from their investigation, rather than just calculating the values. A few candidates considered whether *R* and *S* were exclusive, and provided the contra evidence.

Answers: **(i)** 10/64 **(ii)** 28/64 **(iii)** not independent

Question 4

(i)   The majority of candidates calculated the frequency correctly. A surprising error was stating the class width here.

(ii)   The best solutions presented a table which allowed for the calculation of both the class width and the class frequencies. These were then used to calculate the frequency density within the table. However, some simple arithmetical errors were identified at this stage. Weaker solutions used a continuity correction, which was unnecessary as time is a continuous variable. Good solutions then used scales to ensure that all values necessary to construct were on the grid lines, which resulted in accurate histograms. Common errors were failing to state the units on the horizontal axis, or to label as class width. Weaker solutions often drew either a cumulative frequency graph or a bar chart for either the frequency or cumulative frequency. A number of histograms were seen drawn without a ruler, which is not acceptable at this level.

(iii)   The best solutions continued using the table constructed in **(ii)** to calculate the mid-value, and hence the estimate of the mean. Where a continuity correction had been applied in **(ii)**, there was a resulting accuracy error at this stage. The weakest solutions used the mid-value and the cumulative frequency within their calculations.

Answers: **(i)** 32 **(iii)** 34.7

Question 5

(i)   This question attempted by almost all candidates. The best solutions used a tree diagram, which clarified the possible outcomes and ensured the appropriate probability was identified. More able candidates often simplified the question by considering only camping and non-camping holidays. Weaker solutions did not recognise that this was a conditional probability question, and simple stated P($A \cap C$). A surprising number of simple arithmetical errors were identified within solutions.

(ii)   Where attempted, candidates had a good understanding of the process required. The best solutions stated clearly the required power equation, and then either stated answers calculated from Trial and Improvement or used the properties of logarithms to solve. A number of candidates

penalised themselves by not copying the values carefully from the question, using 0.02 was the most frequent error.

Answers: **(i)** 0.168 **(ii)** 15

Question 6

The context of this question was accessible by all but the weakest candidates. A common error was to use the values for males and females inconsistently through the question.

(i)   The majority of candidates recognised that this was an 'arrangement' question and used permutations correctly, although a few considered $^5P_5$. A number of candidates rounded their exact answer to 3 significant figures. Candidates can be advised that more accurate exact answers are acceptable.

(ii)   The anticipated method was to consider the number of ways each position could be chosen, and then recognising that the values should be multiplied to determine the number of lists. Good candidates often used permutations for the male and female groups before multiplying. A number of candidates considered the combinations for the males and females and then the number of ways these could be arranged.

(iii)   The best solutions identified the different gender combinations that met the stated requirement, and then calculated the separate possibilities before summing. Almost all candidates recognised the key word 'selections' and used combinations appropriately. Some solutions were difficult to follow because the work was not presented in a logical way. A significant minority did not include the group of 5 males and 0 females within their final answer.

(iv)   The best solutions used the same process as in **(iii)** for the 3 remaining places. Common errors were not reducing the number of possible applicants once Mr and Mrs Blake had been chosen, or doing so inconsistently.

Answers: **(i)** 360 360 **(ii)** 5400 **(iii)** 501 **(iv)** 58

Question 7

(i)   Many good solutions contained a sketch of the normal distribution to clarify the information contained within the question. Candidates should be aware that the tables provided include critical values for the normal distribution, which was the anticipated source for the *z*-value. Most solutions used the standardisation formula correctly, with accurate algebraic manipulation to achieve an answer. A few solutions appear to have used a continuity correction, which was not required as time is a continuous variable. There was some confusion as to the area that was being considered, which the use of a sketch could have assisted with.

(ii)   The best solutions correctly interpreted the question as requiring all 4 cars to have taken less than 2 hours to fit. Weaker solutions considered P(>2) and then used this to calculated P(<2). A number of candidates found these values correctly but then failed to complete the question. It was unfortunate that many candidates were unable to gain full credit as they had not worked to a sufficient degree of accuracy in their calculations. Candidates are reminded that to achieve 3 significant figures, then at least 4 significant figures is required throughout and that they need to use a more accurate value for the mean than the acceptable answer in **(i)**.

(iii)   Good solutions developed the method from part **(ii)**. The general standardisation formula was stated, and then appropriate algebraic values substituted. Weaker solutions displayed poor algebraic manipulation, with either the μ being replaced inconsistently by 3σ in the numerator, or the $\mu / 3$ in the denominator not being simplified accurately.

Answers: **(i)** 1.48 **(ii)** 0.758 **(iii)** 0.885

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General comments

In general, candidates scored well on **question 6** whilst **questions 1 and 2** proved more demanding. Candidates were largely able to demonstrate and apply their knowledge in the situations presented, though interpretations of some scenarios proved difficult for some. There was a complete range of scripts from good ones to poor ones.

Most candidates kept to the required level of accuracy, though as is often the case, there were situations where candidates lost marks for giving final answers to less than three significant figure accuracy, this was particularly seen on **question 6(ii)** where errors could have been made because of confusion between 3 s.f. and 3 d.p.

Timing did not appear to be a problem for candidates.

Detailed comments on individual questions follow. Whilst the comments indicate particular errors and misconceptions, it should be noted that there were also some good and complete answers.

Comments on specific questions

Question 1

This was not a well attempted question. Candidates who successfully found E(X) and E(Y) usually went on to successfully find E(X+Y). However, Var(X) and Var(Y) were not always correctly found, and even candidates who did have the correct answers of 25/9 and 5 did not always combine them successfully. A common error was to leave the answer as the variance; the question required the standard deviation.

Answer: 13.3   2.79

Question 2

Candidates were required to set up their Null and Alternative Hypotheses; this was omitted by some candidates. The probability of 18, 19 or 20 hits should then have been calculated using the parameters in the given Binomial situation. A valid comparison was then required (which candidates must clearly show) before the relevant conclusion can be drawn. Errors on this question included attempting invalid approximations, or using incorrect parameters, or merely calculating P(18). It should be noted that the final conclusion drawn should not be written as a 'definite' statement but merely as 'probable' or that 'evidence indicates' etc.

There is no evidence at the 1% level that she has improved

Question 3

Again, Null and Alternative Hypotheses should have been set up at the start of this question. Many candidates successfully found the probability of a Type I error, but common mistakes were to omit √25 when standardising. In **part (ii)**, many candidates successfully explained why a Type II error was not possible.

This question was attempted more confidently than has been previously found with questions on Type I and Type II errors.

Answer: 0.328
H_o *is* rejected but Type II error can only be made if H_o is *not* rejected

Question 4

This was, generally, a well attempted question. The main error seen was in calculating the variance of $X–2Y$; the calculation required was $0.2^2 + 4 \times 0.1^2$ and not $0.2^2 + 2 \times 0.1^2$. Some candidates incorrectly attempted $2X–Y$.

Most candidates successfully standardised with their values for the mean and variance, and chose the appropriate probability.

Answer: 0.362

Question 5

This was a question on which many candidates scored well. Most candidates successfully found unbiased estimates of the mean and variance (though a few candidates found the biased estimate of the variance, and therefore could not gain full marks). Other errors included use of an incorrect 'z' value and it was important that the final answer was written as an interval and not as two separate answers.

In **part (ii)** candidates should have explained that '100' was within the confidence interval before drawing their conclusion. Many candidates realised that a random sample was required to avoid any bias.

Answer: 95.8 to 103
100 lies within the CI, so 'yes'
To avoid bias or to enable statistical inference

Question 6

Candidates demonstrated good knowledge on this question.

In **part (i)** most candidates used the correct value of λ to find the required probability. Some candidates incorrectly interpreted 'fewer than 3 times' and consequently their calculation had an extra unrequired term.

In **part (ii) (a)**, candidates mostly applied the correct approximating distribution (though not all were able to justify this in **part (b)**). Common errors included an incorrect, or no, continuity correction when standardising.

Many candidates found the correct λ in **part (iii)** but, on occasion's, premature approximation led to an incorrect value of λ for some candidates. Similarly, as in **part (i)**, a misinterpretation of 'at least 4' caused errors to be made in this calculation as well.

Answer: 0.481
0.0513
$\lambda > 15$
0.228

Question 7

Many candidates used integration for some parts of this question, rather than using the symmetry of the given diagram. These candidates were often able to reach the required answers, but gave themselves time penalties by using longer methods.

Part (a) was well attempted, though many candidates used an incomplete formula for the variance omitting to subtract $\bar{X}^2$ to reach the required answer. Few candidates were able to 'write down' E(X) and, as mentioned above, attempted a full calculation to reach 0.5. Similarly **part (iii)** was mostly done by integration.

Part (b) was done well by the better candidates. Weaker candidates confused the method for finding the median, and often, incorrectly, equated an integral to 2 rather than using 2 as a limit and equating the integral to 0.5.

Answer: 0.45
0.5
7/27
$2\sqrt{2}$